Statistics in biomedical research, 1^{st} session: Why do we need statistics? Generalities and introduction to the t-test.

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IGH (UMR 9002 CNRS et université de Montpellier)

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This slideshow is accessible at:
http://www.igh.cnrs.fr/equip/Seitz/en_Stats1.pdf

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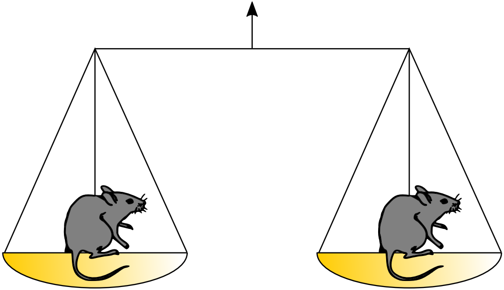

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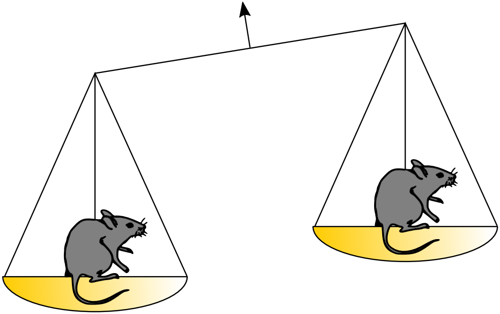

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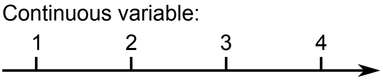

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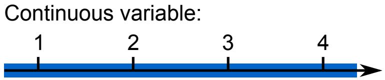

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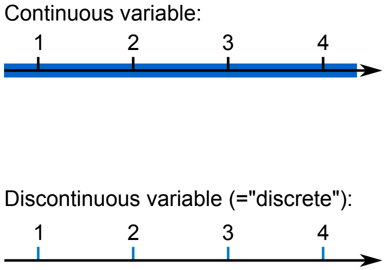

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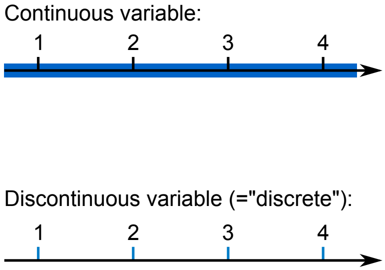

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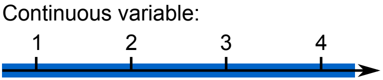

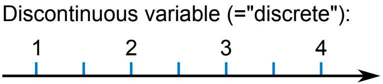

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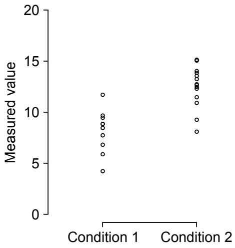

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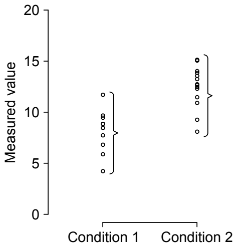

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... but sources of irreproducibility still imply replicating measurements, and comparing intra- to inter-group variability.

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The t-test compares two datasets (example: the same measurement, performed in two experimental conditions, with replicates).

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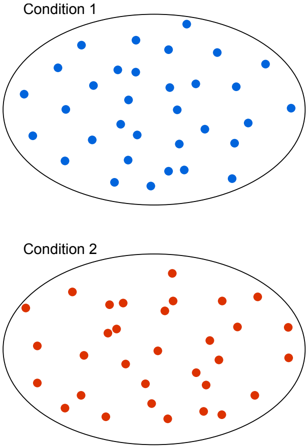

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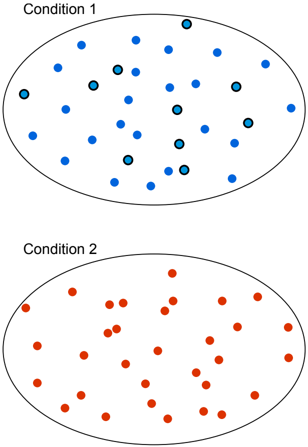

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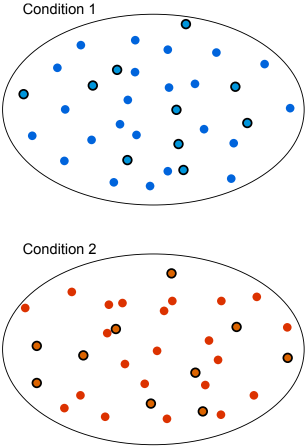

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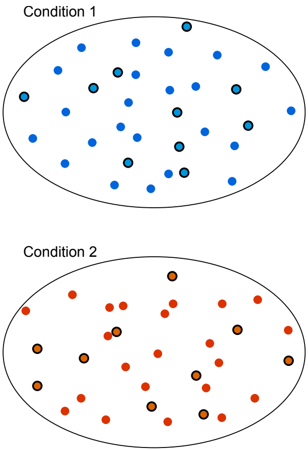

Random picking from an infinite population.

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$\longrightarrow$ The t-test permits concluding on the equality of means of infinite cohorts (inaccessible to experimentation).

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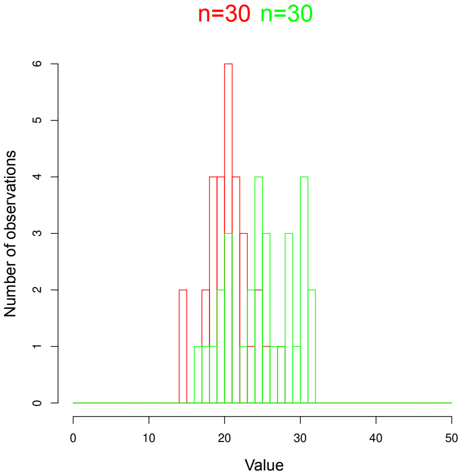

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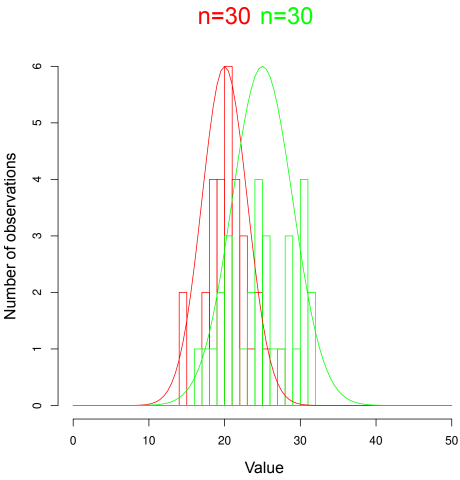

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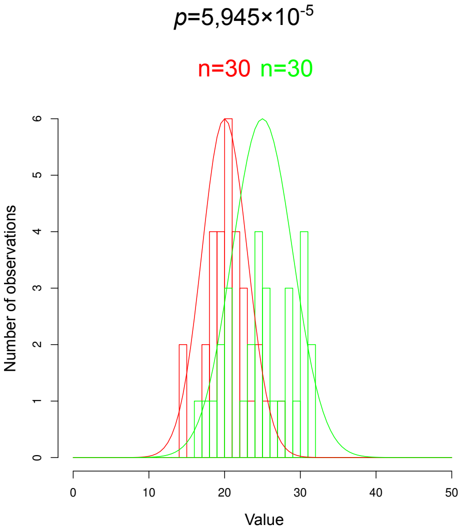

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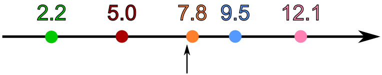

$$m = \frac{2.2+5.0+7.8+9.5+12.1}{5} = 7.32$$

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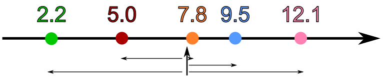

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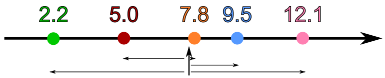

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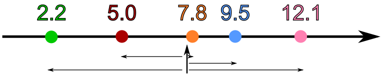

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$$v_{samp.} = \frac{(7.32-2.2)^2+(7.32-5.0)^2+(7.8-7.32)^2+(9.5-7.32)^2+(12.1-7.32)^2}{4} = 14.857$$

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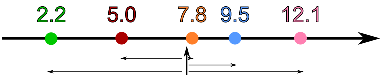

$$\sigma_{pop.} = \sqrt{V_{pop.}} = 3.44755$$

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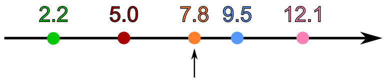

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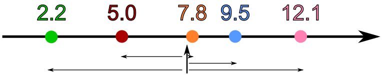

$$CV_{pop.} = \frac{\sigma_{pop.}}{m_{pop.}}$$

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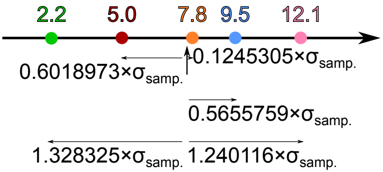

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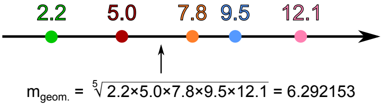

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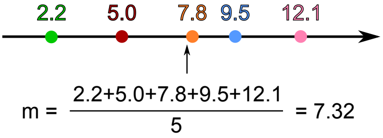

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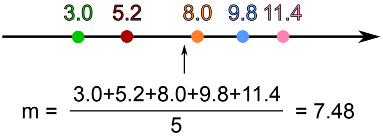

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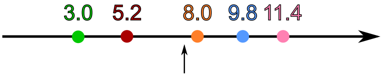
- Geometric mean: n^{th} root of the product of all n values.
- Standard error (implicitly: "of the mean"): standard deviation of a distribution of estimations of the mean (based on distinct sampled datasets). Calling σ the (theoretically identical) standard deviation of each of these sampled datasets, standard error equals: $\frac{\sigma}{\sqrt{n}}$.



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Graphical display

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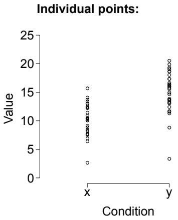

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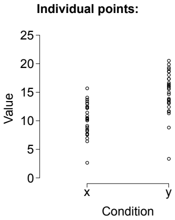

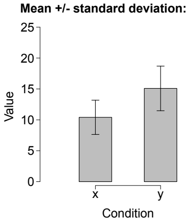

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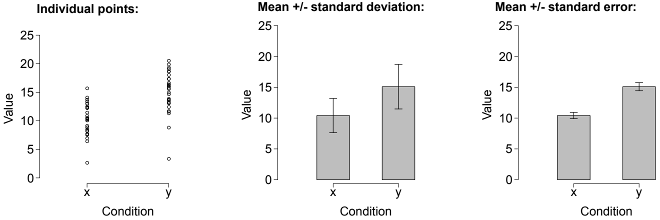

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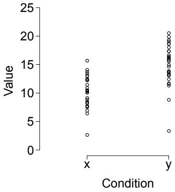
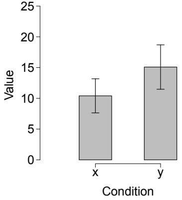
Individual points:



Mean +/- standard deviation:



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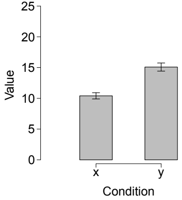
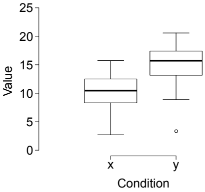


Boxplot:



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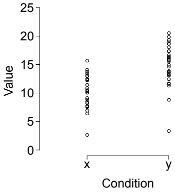

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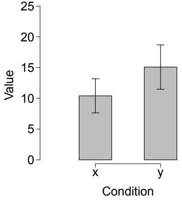

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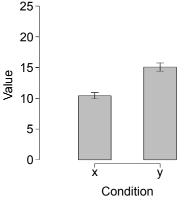

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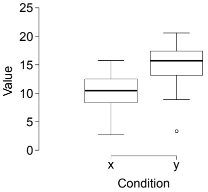
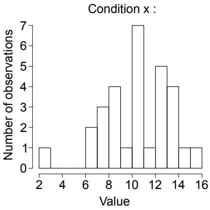


Histogram:



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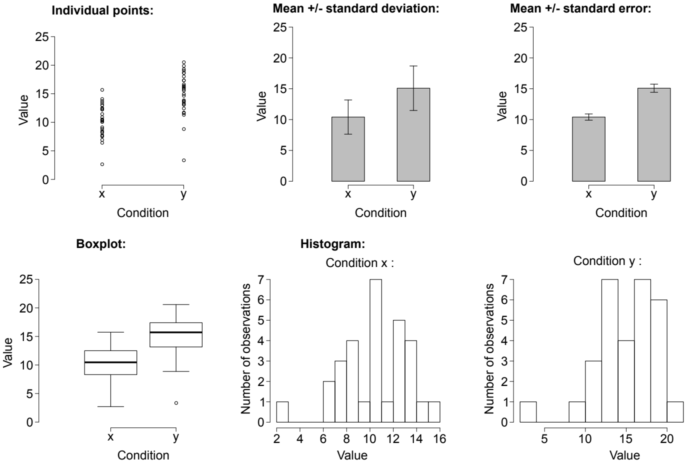

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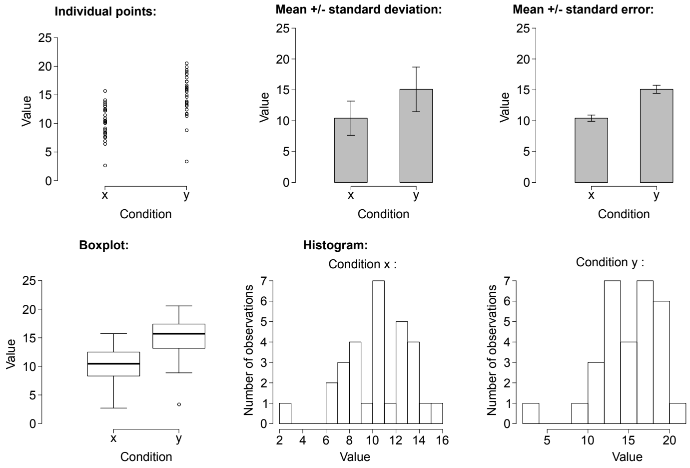

Always explain what error bars represent!